The Previous Analysis was for **Any Iterative Search Method**. We applied the analysis to Random Search and Greedy Search.

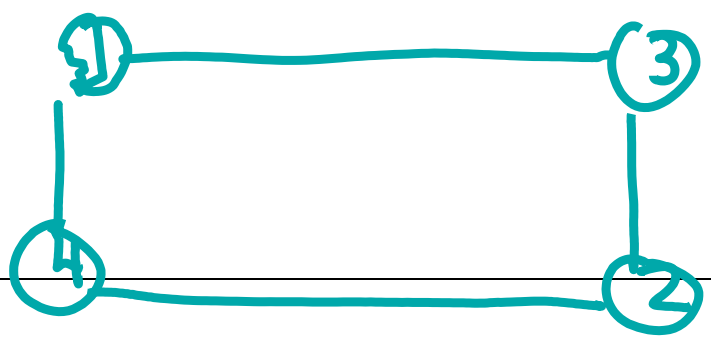
Now we move to a more detailed analysis of Simulated Annealing (in chapter 2)

- What is the acceptance $A_{ij}(T)$ for simulated Annealing with a fixed parameter T?

- What is the value of the matrix Θ?

- What can we say about the convergence of Simulated Annealing?

- Let's look first at an example from the book.



4 states
Cost $i = i$

Neighbors as drawn

SIMULATED ANNEALING

Assume we have a scalar optimization problem with only 4 possible values for the decision variable. The cost of the 4 values is given inside the circle. The neighborhood is just adjacent points. What is the configuration graph for a simulated annealing search?

How do we work this out? We assume equal probability of picking any of the neighbors. I.e. $P_{12} = 1/2$, $P_{14} = 1/2$ $P_{13} = 0$, because 3 isn't a neighbor

Simulated Annealing

- There are two steps in SA:
- 1) Pick the neighbor whose cost function you will evaluate= *"picking a neighbor"*
- *2)* Deciding if you will accept the move to that neighbor = *"accepting the neighbor"*
- Probability of *"picking a neighbor"*= p_{kj}
- Probability of *"accepting the neighbor"*=A_{kj}

So total probability is product of p and A

So let θ_{ij} be the probability of going from I to j

$$\theta_{ij}(T) = \begin{cases} A_{ij}(T)\, p_{ij} & \text{if } i \neq j \\ 1 - \sum_{i \neq k} A_{ik}(T)\, p_{ij} & \text{if } i = j \end{cases}$$

→ Temperature

small t = time (iteration)

T is the iteration varying parameter (e.g. temperature)

What is A_{kj} for SA?

- $(Cost(j) - Cost(k))$ =(def)= $\Delta Cost_{kj}$
- So the probability of accepting an uphill move from k to j is
- = exp (- $\Delta Cost_{kj}$)
- Now when we consider the transition probabiltiy θ, there are 4 situations
  - Downhill move
  - Uphill move accepted
  - No move
  - No edge so no possibility to move from k to j

enumerated next set of slides.